Pulse propagation and optical-klystron free-electron-laser devices

G. Dattoli, L. Mezi, and L. Bucci
ENEA, Dipartimento Innovazione, Divisione Fisica Applicata, Centro Ricerche Frascati, Casella Postale 65, 00044 Frascati, Rome, Italy
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We derive the equation governing the evolution of the optical field amplitude of a free-electron-laser oscillator operating in the optical-klystron configuration. The equation includes short pulse effects, due to the finite length of the electron bunches, and are valid under the assumption of small signal regime and low gain. Stationary solutions of the supermode type, analogous to those of conventional free-electron-laser configurations, are shown to exist for this case, too. Analytical expressions for the optical pulse shape are derived, without any assumption of dominance of the dispersive section, in the case of the long bunch regime, namely when the bunch length is larger than the slippage length. We present useful gain formulas, including the combined effect of dispersive section and pulse effects, and discuss the limits of validity of our treatment.

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I. INTRODUCTION

An optical-klystron (OK) free-electron laser (FEL) employs two undulators (usually with the same number of periods N_u and period length λ_u), separated by a drift section (see Fig. 1). A device of this type is used to increase the gain when long straight sections are not available, as in the case of the FEL operating with storage rings. The mechanism leading to the gain enhancement is simply due to the fact that in the first undulator an *e*-beam energy modulation occurs; such a modulation transforms into a density modulation in the drift part thus providing a more substantive emission in the second undulator. The crucial factor, characterizing the increment of gain with respect to a device operating in the normal configuration, is the ratio between the length of the drift section to the individual undulator length. This quantity denoted by δ (see below) controls much of the physics of the system and must be carefully chosen to avoid gain reduction effects due to inhomogeneous broadening contributions or limitations in the attainable output power (for further comments see Ref. [1]).

In most of the OK-FEL experiments [2,3] values of δ significantly larger than unity have been chosen. This type of device has operated so far with storage rings, characterized by an electron beam with an extremely low energy spread so that the constraints due to inhomogeneous contribution can be ignored.

The assumption of large δ allows a noticeable simplification of the relevant gain expressions and of the evaluation of the quantities characterizing the evolution of the system. It may happen, however, that the optimization of the device indicates that it is convenient to operate with values of δ not too large with respect to unity, this may occur if it is convenient to operate at larger output power or in situations in which some kind of instability is affecting the quality of the beam as it happens in high current storage rings. In this hypothesis one cannot perform simplifying assumptions and is obliged to deal with the complete gain expression.

In this paper we will treat the problem of pulse propagation in OK FEL's without the assumption of a dominant dispersive section length. The results we will obtain complete those obtained by Elleaume [4] and Litvinenko and Vinokurov [5] and provide useful scaling laws concerning the effect of short pulses on the OK FEL performances.

The gain of free-electron-laser (FEL) devices reads [6,1]

$$G(\nu,\delta)=-2\pi g_0\frac{\partial}{\partial\nu}\times\left[\left(\frac{\sin\left(\frac{\nu}{2}\right)}{\frac{\nu}{2}}\right)^2\{1+\cos[\nu(1+\delta)]\}\right],$$

$$\delta=\frac{N_d}{N_u}. \tag{1}$$

The quantities g_0 and ν denote the small signal gain coefficient associated with one undulator section and the detuning parameter, respectively, and are linked to the device parameter by (mks units are used, N_d is the number of equivalent periods of the dispersive section),

$$g_0=\frac{16\pi}{\gamma}\lambda_r L_u\frac{J}{1.7\times10^4}N_u^2\xi f_b(\xi),$$

$$\xi=\frac{1}{4}\frac{k^2}{1+\frac{k^2}{2}},\quad f_b(\xi)=J_0(\xi)-J_1(\xi),$$

$$\lambda_r=\frac{\lambda_u}{2\gamma^2}\left(1+\frac{k^2}{2}\right),\quad L_u=N_u\lambda_u,$$

$$\omega_r=\frac{2\pi c}{\lambda_r},\quad \nu=2\pi N_u\frac{\omega_r-\omega}{\omega_r}, \tag{2}$$

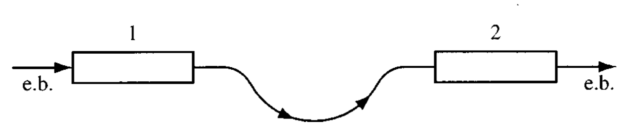

FIG. 1. Layout of an optical klystron device.

where k is the undulator strength and $f_b(\,)$ is the Bessel factor correction due to the assumption that the undulators are linearly polarized. The physical meaning of Eq. (1) is fairly transparent; it contains indeed the gain contribution from the two separated undulators and the gain enhancing part due to the interference contribution.

For reasons that will appear more clearly in the following, it is convenient to recast Eq. (1) as

$$G(\nu,\delta)=-2\pi g_0\frac{\partial}{\partial\nu}\Bigg[4\left(1+\frac{\delta}{2}\right)^2\left(\frac{\sin(\overline{\nu})}{\overline{\nu}}\right)^2+\delta^2\left(\frac{\sin\left(\frac{\bar{\bar{\nu}}}{2}\right)}{\frac{\bar{\bar{\nu}}}{2}}\right)^2+2\left(\frac{\sin\left(\frac{\nu}{2}\right)}{\frac{\nu}{2}}\right)^2-2\left(\frac{\sin\left(\frac{\tilde{\nu}}{2}\right)}{\frac{\tilde{\nu}}{2}}\right)^2(1+\delta)^2\Bigg], \tag{3}$$

where we have set

$$\overline{\nu}=\nu\left(1+\frac{\delta}{2}\right),\quad \bar{\bar{\nu}}=\nu\delta,\quad \tilde{\nu}=\nu(1+\delta). \tag{4}$$

Equation (3) has the advantage of being written as four independent gain contributions. Any one of Eqs. (1)–(3) account for the absorptive part of the interaction only. However Eq. (3) suggests that the inclusion of the dispersive part can be achieved by defining the following complex gain function:

$$\Gamma(\nu,\delta)=-2g_0\frac{\partial}{\partial\nu}\left[4\left(1+\frac{\delta}{2}\right)^2h(2\overline{\nu})+\delta^2h(\bar{\bar{\nu}})+2h(\nu)-2h(\tilde{\nu})(1+\delta)^2\right], \tag{5}$$

where

$$h(\nu)=2\pi\int_0^1(1-t)e^{i\nu t}dt. \tag{6}$$

The real and imaginary parts of Eq. (5) account for the absorptive and dispersive parts of the interaction, respectively.

It must be underlined that the analysis developed so far applies to devices operating with a continuous e-beam. Pulse propagation effects arise in devices operating with radio-frequency accelerators, which provide e-beams with a pulsed structure reflecting on the laser structure itself. We will assume in the following that electronic packets are characterized by longitudinal distributions $f(z)$ with rms length σ_z.

The problems associated with short pulse effects have been widely discussed in FEL literature [7–9], and will not be reconsidered here. In full analogy with the ordinary FELs [7] and according to the definition of complex gain given in Eq. (5), we write the OK-FEL low gain pulse propagation equation as

$$2T_c\frac{\partial}{\partial t}a(z,t)+\eta a(z,t)+\Delta g_0\Theta\frac{\partial}{\partial z}a(z,t)=-2i\frac{(2\pi)^{3/2}g_0}{\mu_c\Delta^2}\int_0^\Delta dy\,yQ\left(\nu\frac{y}{\Delta},\delta\right)\times a(z+y,t)\int_{z+y}^{z+\Delta}f(\tilde{z})d\tilde{z}, \tag{7}$$

where t is a discrete time linked to the cavity round trip period T_c, $a(z,t)$ is the Colson's field complex amplitude, η denotes the cavity losses, $\Delta=N\lambda$ is the slippage distance over one undulator length, $\mu_c=\Delta/\sigma_z$ is the coupling parameter, and [see also Eqs. (5) and (6)]

$$Q(\nu,\delta)=8\left(1+\frac{\delta}{2}\right)^3e^{2i\nu[1+(\delta/2)]}+\delta^3e^{i\nu\delta}+2[e^{i\nu}-(1+\delta)^3e^{i\nu(1+\delta)}]. \tag{8}$$

The parameter Θ is linked to the cavity mismatch δL_c, necessary to compensate the lethargic effect, by the relation

$$\Theta=\frac{4\,\delta L_c}{g_0\Delta}. \tag{9}$$

The physical meaning of Eq. (7) is fairly straightforward; it contains indeed all the characteristic effects of pulse propagation dynamics.

The laser field value at a given point preserves the memory of the gain process in one pass. The convolution integral on the right-hand side (rhs) of Eq. (7) contains the effect of the relative slippage, between electron and optical packets, and the relevant gain contribution weighted with the contributing part of the electron packet. Equation (7) cannot be solved analytically as it stands; useful analytical solutions can be obtained under physical assumptions concerning the length of the electron bunch compared to the slippage distance. This aspect of the problem and the relevant results will be discussed in the forthcoming sections.

II. THE OK-FEL PULSE PROPAGATION EQUATION IN THE LONG BUNCH APPROXIMATION

It has been shown that, in the case of ordinary FELs, the integro-differential equation (7) can be transformed into an exactly solvable second-order Fokker-Planck equation if the electron bunch length is much larger than the slippage length [8,9], namely,

$$\Delta\ll\sigma_z. \tag{10}$$

This assumption implies that the optical field has a slowly varying amplitude experiencing a small portion of the electron bunch around the maximum of its distribution. We can therefore make an expansion in the longitudinal coordinate, approximate the electron bunch distribution as

$$f(z)\cong\frac{1}{(2\pi)^{1/2}\sigma_z}\left(1-\frac{z^2}{2\sigma_z^2}\right), \tag{11}$$

and finally get

$$\frac{\partial}{\partial\tau}a(\tilde{z},\tau)=\left[\Omega_2\left(\frac{1}{2}\frac{\partial^2}{\partial\tilde{z}^2}\right)+\Omega_1\left(\frac{1}{2}\tilde{z}^2\right)+\Omega_4\frac{\partial}{\partial\tilde{z}}+\Omega_3\tilde{z}+\Omega_5\right]a(\tilde{z},\tau),$$

$$\tilde{z}=\frac{z}{\sigma_z},\quad \tau=\frac{g_0 t}{2T_c}, \tag{12}$$

where we have set

$$\Omega_1=-\Gamma_1(\nu,\delta),\quad \Omega_2=\mu_c^2\Gamma_3(\nu,\delta),$$

$$\Omega_3=-\frac{\mu_c}{2}[1+\Gamma_1(\nu,\delta)],\quad \Omega_4=\mu_c[\Gamma_2(\nu,\delta)-\Theta],$$

$$\Omega_5=\Gamma_1(\nu,\delta)-\frac{\eta}{g_0} \tag{13}$$

and

$$\Gamma_1(\nu,\delta)=-2\frac{\partial}{\partial\nu}H(\nu,\delta),$$

$$\Gamma_2(\nu,\delta)=2i\frac{\partial^2}{\partial\nu^2}H(\nu,\delta),$$

$$\Gamma_3(\nu,\delta)=2\frac{\partial^3}{\partial\nu^3}H(\nu,\delta),$$

$$H(\nu,\delta)=4\left(1+\frac{\delta}{2}\right)^2 h(2\overline{\nu})+\delta^2 h(\bar{\bar{\nu}})+2h(\nu)-2h(\overline{\nu})(1+\delta)^2. \tag{14}$$

The pulse propagation equation has been therefore reduced to a diffusive equation with a quadratic ''potential.'' The eigenstates associated with Eq. (11) are the OK-FEL supermodes (SM) with eigenvalues [8]

$$\lambda_n=\Gamma_1(\nu,\delta)-\frac{\eta}{g_0}-\frac{[\Gamma_2(\nu,\delta)-\Theta]^2}{2\Gamma_3(\nu,\delta)}-\mu_c\left(n+\frac{1}{2}\right)\times[\Gamma_1(\nu,\delta)\Gamma_3(\nu,\delta)]^{1/2}. \tag{15}$$

The physical meaning of the so far obtained results will be discussed below.

The real part of λ_n is linked to the OK-FEL SM gain, while the imaginary part to its phase variation. It is worth noting that (a) the first two terms provide essentially the usual gain-loss contribution (note that $\mathrm{Re}[\Gamma_1(\nu,\delta)]=G(\nu,\delta)/g_0$, (b) the third term provides a contribution due to the lethargy and cavity mismatch, and (c) the last term removes the degeneracy between the various supermodes and is linked to the coupling parameter μ_c which controls the slippage effects and the strength of the coupling between the longitudinal modes [7].

It has been shown that apart from the higher-order correction in δ the maximum continuous beam OK-FEL gain

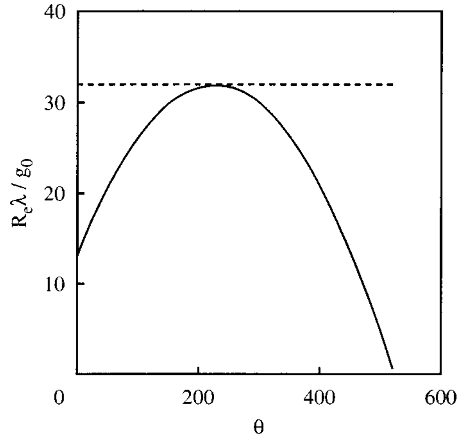

FIG. 2. $\mathrm{Re}(\lambda_0)/g_0$ vs Θ for $\delta=5$, $\mu_c=0.05$, $\nu^*=0.26$ (continuous line) maximum gain evaluated from Eq. (19).

[namely, the real part of $\Gamma_1(\nu^*,\delta),v^*$ is the value of the detuning parameter where the maximum gain value is located] scales with δ as [1]

$$\mathrm{Re}[\Gamma_1(\nu^*,\delta)]=8\times0.85\times(1+0.913\delta). \tag{16}$$

An idea of the corrections due to the other terms are provided by Fig. 2 where we have reported the gain of the fundamental $(n=0)$ SM vs Θ. It is worth noting that the maximum gain is obtained for

$$\mathrm{Re}[\Gamma_2(\nu^*,\delta)]=\Theta. \tag{17}$$

We have numerically checked that

$$\mathrm{Re}[\Gamma_2(\nu^*,\delta)]\cong0.456\times2\times8\times(1+0.913\delta)^2. \tag{18}$$

Combining this last result and that relevant to the definition of Θ, we obtain the following value of the cavity length mismatch compensating the lethargic effect:

$$\delta L_c=\tfrac{1}{4}(0.456 g_{\mathrm{OK}}\Delta_{\mathrm{OK}}),$$

$$g_{\mathrm{OK}}=8g_0(1+0.913\delta),$$

$$\Delta_{\mathrm{OK}}=2\Delta(1+0.913\delta). \tag{19}$$

Equation (18) reproduces the result of the ordinary FEL, provided that the small signal gain coefficient is replaced by g_{OK} and the slippage by Δ_{OK} which represents the total slippage including the contribution of the dispersive section.

The effect on the gain of the third contribution is that of providing a further reduction; the inclusion of this term yields for the fundamental supermode a perfectly matched cavity and negligible losses,

$$g_0\,\mathrm{Re}(\lambda_0)\cong0.85g_{\mathrm{OK}}\left(1-\frac{\mu_c^{\mathrm{OK}}}{x(\delta)}\right),$$

$$\mu_c^{\mathrm{OK}}=\frac{\Delta_{\mathrm{OK}}}{\sigma_z}, \tag{20}$$

where $x(\delta)$ is a function exhibiting an almost linear dependence vs δ and can be parametrized as

$$x(\delta)\cong 3+0.1\delta. \tag{21}$$

The results of this section yield an idea of what the impact of pulse propagation effects are on the OK-FEL gain; further and more quantitative considerations will be developed in the next section.

III. CONCLUDING REMARKS

We must underline that the so far developed considerations apply, e.g., to the FEL experiment in progress at Trieste with Elettra Ring, all the relevant parameters can be found in Ref. [10]. Here we will discuss the case of a FEL operating with a high energy Linac having an e-beam energy of 200 MeV, an undulator period length $\lambda_u=15$ cm, undulator strength $K=\sqrt{2}$, a number of periods of each undulator $N=10$, $\delta=10$, an electron pulse with $\sigma_z=10^{-2}$ cm, and a small signal gain coefficient $g_0=5\times 10^{-3}$. In this case the net gain of the device is given in Fig. 3; the constant dotted line refers to the total gain in absence of pulse propagation corrections and the dashed curve refers to the gain of the SM of order 1. The optimum cavity mismatch can be calculated from Eq. (19) and is about 1 μm.

We have already remarked that the present analysis holds under a number of approximation that is worth summarizing: (a) the integral equation can be derived under the assumption of low gain, which implies that the OK small signal coefficient g_{OK} should satisfy the condition

$$g_{\text{OK}}<0.3, \tag{22}$$

and (b) the expansion leading to Eq. (12) imposes a condition of the type (9), involving the total slippage, namely,

$$\Delta_{\text{OK}}\leqslant\sigma_z. \tag{23}$$

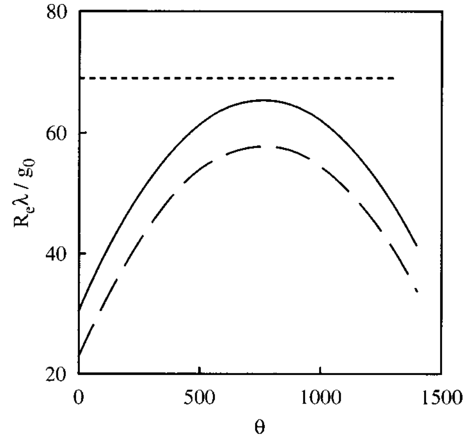

FIG. 3. Maximum gain of an OK FEL without pulse propagation corrections (dotted line). Gain of the zeroth-order SM vs Θ (continuous line). Gain of the first-order SM vs Θ (dashed line). $\nu^*=0.15$, $\delta=10$, $\mu_c=0.01$.

To be more quantitative we can fix the limit of validity of the present analysis to $\mu_c^{\text{OK}}\leqslant 0.5$.

Regarding the nature of the OK-FEL-SM we note that they are essentially harmonic-oscillator eigenfunctions, as suggested by the fact that Eq. (12) is a Schrödinger-like equation with a quadratic potential. Without entering into the details of the computation which can be found in Refs. [8], [9], we write the SM as

$$\Phi_n(\tilde{z})=H_n[a(\tilde{z}+b)]\exp[-c(\tilde{z}+d)^2], \tag{24}$$

where $H_n(\cdots)$ are Hermite polynomials and (a,b,c,d) are expressed in terms of the Γ functions, namely,

$$a=\frac{1}{\sqrt{\mu_c}}\left(\frac{\Gamma_1(\nu,\delta)}{\Gamma_3(\nu,\delta)}\right)^{1/4},\quad c=\frac{1}{2\mu_c}\left(\frac{\Gamma_1(\nu,\delta)}{\Gamma_3(\nu,\delta)}\right)^{1/2},\quad b=\frac{\mu_c[\Gamma_1(\nu,\delta)+\Gamma_2(\nu,\delta)]}{2\Gamma_1(\nu,\delta)},$$

$$d=\frac{\mu_c[\Gamma_1(\nu,\delta)+\Gamma_2(\nu,\delta)]+2\sqrt{\Gamma_1(\nu,\delta)/\Gamma_3(\nu,\delta)}[\Gamma_3(\nu,\delta)-\Theta]}{2\Gamma_1(\nu,\delta)}. \tag{25}$$

The rms width of the fundamental SM is

$$\tilde{\sigma}=\frac{\sqrt{\mu_c}}{\sqrt{2\,\text{Re}[\Gamma_1(\nu,\delta)/\Gamma_3(\nu,\delta)]}}. \tag{26}$$

Recalling that $\tilde{z}=z/\sigma_z$, we find the following scaling relation:

$$\sigma=f(\delta)\sqrt{\Delta_{\text{OK}}\sigma_z}, \tag{27}$$

where $f(\delta)$ is a function which roughly scales as

$$f(\delta)\cong 1+0.07\delta. \tag{28}$$

Equation (26) yields an idea of the length of the light pulse. Further comments on the nature of the OK-FEL-SM will be presented in a forthcoming paper.

We must underline that the results obtained in this paper are not dissimilar from analogous theoretical estimations [4,5]. The present analysis is more general since it does not contain the assumption that the OK part of the gain is dominating and is general enough to relax condition (10).

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